



# Optimisation de forme pour la conception des cavités de liner

Gilles Tissot, Gwénaél Gabard

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# Shape optimisation for acoustic liners design

Gilles TISSOT, Gwénaél GABARD

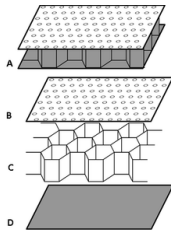
Laboratoire d'Acoustique de l'Université du Mans

Congrès Français d'Acoustique  
Le Havre, Wednesday, 25 April 2018



# Context

## Acoustic liners:



Schematic  
representation.



Integrated in  
nacelles of aircrafts.

## Constraints:

- Low frequency absorption requires deep cavities.
- Challenge to push away this limitation.

*MACIA* ANR project: LAUM, SAFRAN.

# Context

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## Objective:

- Optimise cavity shape of liners.
- Make it resonate at lower frequencies.

## Strategy:

- Shape optimisation.
- Based on frequency response.

# Outline

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- 1 Governing equations
- 2 Shape optimisation
- 3 Numerical implementation
- 4 Results

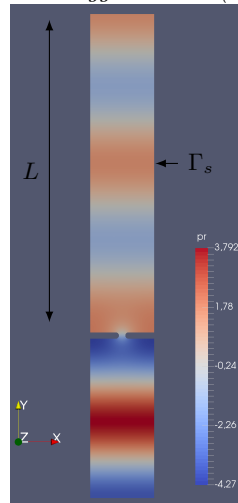
# Governing equations

Helmholtz equation:

$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 & \text{on } \Gamma_s \text{ (free slip)}. \end{cases}$$

*Bossart et al. (2003)*

*Berggren et al. (2018)*



# Governing equations

Helmholtz equation:

$$\left\{ \begin{array}{l} \Delta p + k^2 p = 0 \quad \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_s \text{ (free slip)} \\ \nabla p \cdot \mathbf{n} = \delta_v \frac{i-1}{2} \Delta_T p - \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} p \\ \quad \quad \quad \text{on } \Gamma_w \text{ (walls)}. \end{array} \right.$$

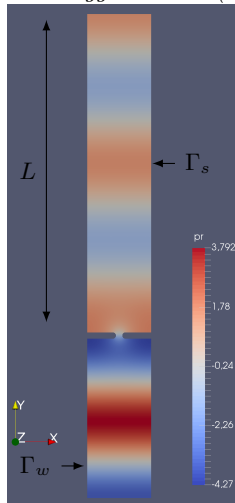
$$\bullet \delta_v = \sqrt{\frac{2}{\omega Re_a}} \quad ; \quad \delta_T = \sqrt{\frac{2}{\omega Re_a Pr}}.$$

$$\bullet \nabla f = \nabla_T f + \frac{\partial f}{\partial n} \mathbf{n} ;$$

$$\Delta f = \Delta_T f + \frac{\partial^2 f}{\partial n^2} + \nabla_T \cdot \mathbf{n} \frac{\partial f}{\partial n}$$

*Bossart et al. (2003)*

*Berggren et al. (2018)*



# Governing equations

Helmholtz equation:

$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 & \text{on } \Gamma_s \text{ (free slip)} \\ \nabla p \cdot \mathbf{n} = \delta_v \frac{i-1}{2} \Delta_T p - \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} p & \text{on } \Gamma_w \text{ (walls)} \\ \nabla p \cdot \mathbf{n} + ikp = 2ike^{ikL} & \text{on } \Gamma_Z \text{ (Impedance B.C.).} \end{cases}$$

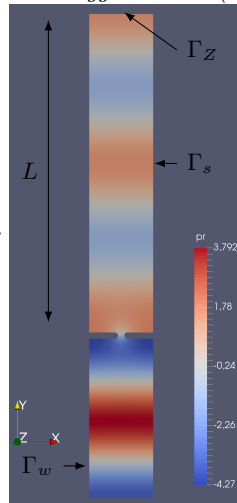
- Unitary amplitude of incoming plane wave.
- Reflexion coefficient:

$$R = \left( \frac{1}{L_Z} \int_{\Gamma_Z} p(x, L) dx - e^{ikL} \right) e^{ikL}.$$

- Impedance:  $Z = \frac{1 + R}{1 - R}$ .

*Bossart et al. (2003)*

*Berggren et al. (2018)*





# Governing equations

Helmholtz equation:

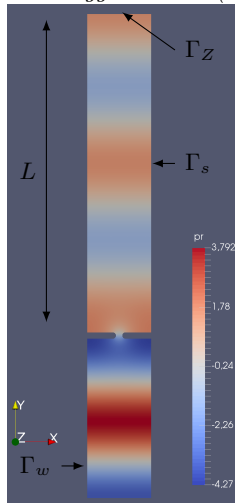
$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 & \text{on } \Gamma_s \text{ (free slip)} \\ \nabla p \cdot \mathbf{n} = \delta_v \frac{i-1}{2} \Delta_T p - \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} p & \text{on } \Gamma_w \text{ (walls)} \\ \nabla p \cdot \mathbf{n} + ikp = 2ike^{ikL} & \text{on } \Gamma_Z \text{ (Impedance B.C.).} \end{cases}$$

- Modify cavity shape for improving behaviour: *Target impedance*.

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

Bossart et al. (2003)

Berggren et al. (2018)

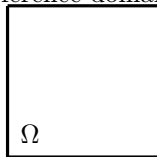


# Shape optimisation

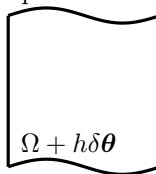
## Generalities

Infinitesimal domain variations:

Reference domain:



Shape variation:



$h$  small.

# Shape optimisation

## Generalities

Cost functional: **unconstrained** minimisation problem *Allaire (2003,2004)*

$$\mathcal{J}(\Omega) = \int_{\Omega} f(x) \, dx$$

Shape derivative:  $\forall \delta \boldsymbol{\theta}$  we have

$$(\nabla_{\delta \boldsymbol{\theta}} \mathcal{J}, \delta \boldsymbol{\theta}) = \lim_{h \rightarrow 0} \frac{\mathcal{J}(\Omega + h \delta \boldsymbol{\theta}) - \mathcal{J}(\Omega)}{h} = \int_{\Gamma_{\theta}} \delta \boldsymbol{\theta}(x) \cdot \boldsymbol{n}(x) f(x) \, dx.$$

# Shape optimisation

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Descent direction:

$$\mathcal{J}(\Omega + h \delta \boldsymbol{\theta}) = \mathcal{J}(\Omega) + h \int_{\Gamma_{\theta}} \delta \boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx + \mathcal{O}(h^2)$$

# Shape optimisation

## Generalities

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Descent direction:

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$\Rightarrow \delta \boldsymbol{\theta}(x) = -f(x) \mathbf{n}(x)$  ensures a descent direction!

Indeed: 
$$\mathcal{J}(\Omega + h \delta \boldsymbol{\theta}) = \mathcal{J}(\Omega) - h \int_{\Gamma_{\theta}} f^2(x) \, dx + \mathcal{O}(h^2)$$

# Shape optimisation

## Impedance matching

*Bängtsson et al. (2003)*

Cost functional:

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

Impedance:  $Z(p, k) = \frac{e^{-ikL} - e^{ikL} + \bar{p}}{e^{-ikL} + e^{ikL} - \bar{p}}$  with  $\bar{p} = \frac{1}{L_Z} \int_{\Gamma_Z} p \, dl.$

# Shape optimisation

## Impedance matching

Bängtsson et al. (2003)

Cost functional:

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subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

We define the Lagrangian (*constrained*  $\mapsto$  *unconstrained*)

$$\mathcal{L}(\Omega, p, \lambda) = \mathcal{J}(\Omega, p) - \text{real} \left( \int_{k_1}^{k_2} (\lambda, \Delta p + k^2 p)_\Omega dk \right)$$

# Shape optimisation

## Impedance matching

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Derivative with respect to each variable:

$$(\nabla_{\delta\lambda} \mathcal{L}, \delta\lambda) = (\nabla_{\delta p} \mathcal{L}, \delta p) = 0$$

$$(\nabla_{\delta\theta} \mathcal{L}, \delta\theta) = 0 \Rightarrow (\nabla_{\delta\theta} \mathcal{J}, \delta\theta)$$



# Impedance matching

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Derivative with respect to  $\lambda$ :  $\Delta p + k^2 p = 0$ .

# Impedance matching

Derivative with respect to  $\lambda$ :  $\Delta p + k^2 p = 0$ .

Derivative with respect to  $p$ : Adjoint equation

$$\left\{ \begin{array}{ll} \Delta \lambda + (k^*)^2 \lambda = 0 & x \in \Omega \\ \nabla \lambda \cdot \mathbf{n} = 0 & x \in \Gamma_s \\ \nabla \lambda \cdot \mathbf{n} + \delta_v \frac{i+1}{2} \Delta_T \lambda - \delta_T (k^*)^2 \frac{(i+1)(\gamma-1)}{2} \lambda = 0 & x \in \Gamma_w \\ \nabla \lambda \cdot \mathbf{n} - ik^* \lambda = \frac{2}{L_Z} \frac{e^{ikL} (Z(p, k) - Z_T(k))}{\left( (e^{-ikL} + e^{ikL} - \bar{p})^2 \right)^*} & x \in \Gamma_Z. \end{array} \right.$$

# Impedance matching

Derivative with respect to  $\lambda$ :  $\Delta p + k^2 p = 0$ .

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Derivative with respect to  $\Omega$ : For any displacement direction  $\delta \theta$ ,

$$(\nabla_{\delta \theta} \mathcal{J}, \delta \theta) = \int_{\Gamma_\theta} (\delta \theta(x) \cdot \mathbf{n}(x)) \int_{k_1}^{k_2} \text{Real} [-\nabla \lambda^* \cdot \nabla p + k^2 \lambda^* p] dk dx.$$

# Impedance matching

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## Algorithm:

Solve iteratively:

- Solve Helmholtz direct  $\Rightarrow p(k)$  (*frequency response*).
- Solve Helmholtz adjoint  $\Rightarrow \lambda(k)$  (*frequency response*).
- Optimality condition  $\Rightarrow \nabla_{\delta\theta} \mathcal{J}$ .
- Move shape with linesearch algorithm (*Armijo backtracking*).
- Iterate until convergence.

# Numerical implementation

*Moës et al. (1999)*

Finite elements with immersed boundary:

*Osher et al. (2001)*

- **XFEM** for integration methods on cut elements (GETFEM++).

*Protas et al. (2004)*

- **Level set:**

$$\begin{cases} \psi = 0 & x \in \Gamma_\theta \\ \psi > 0 & x \in \Omega \\ \psi < 0 & x \notin \Omega. \end{cases}$$

- Transport equation for moving the shape:

$$\frac{\partial \psi}{\partial t} + v \mathbf{n} \cdot \nabla \psi = 0.$$

with

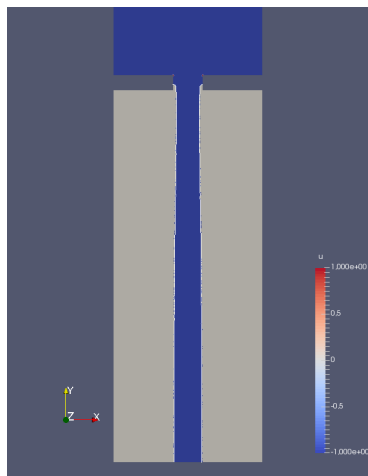
$$v = -\nabla_{\delta \theta} \mathcal{J} \cdot \mathbf{n}. \quad (\text{Regularised by Sobolev Gradient})$$

$\mathbf{n}$  is the outward wall normal direction. SUPG discretisation.

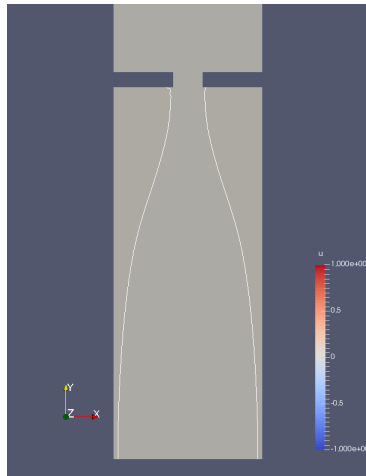


# Results

No viscosity:  $Z_T = 0$ ,  $k = [0.2 : 0.6]$



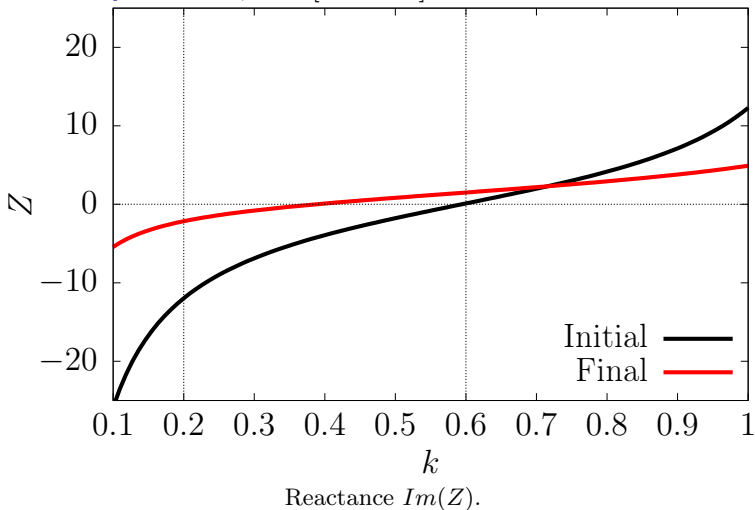
Initial condition.



Converged.

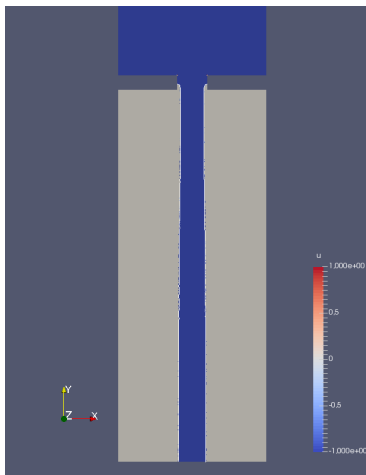
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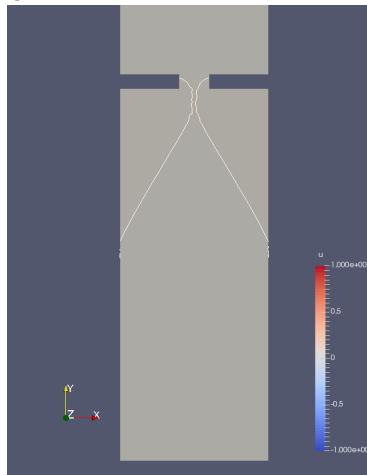


# Results

No viscosity:  $Z_T = 0$ ,  $k = [0.15 : 0.25]$



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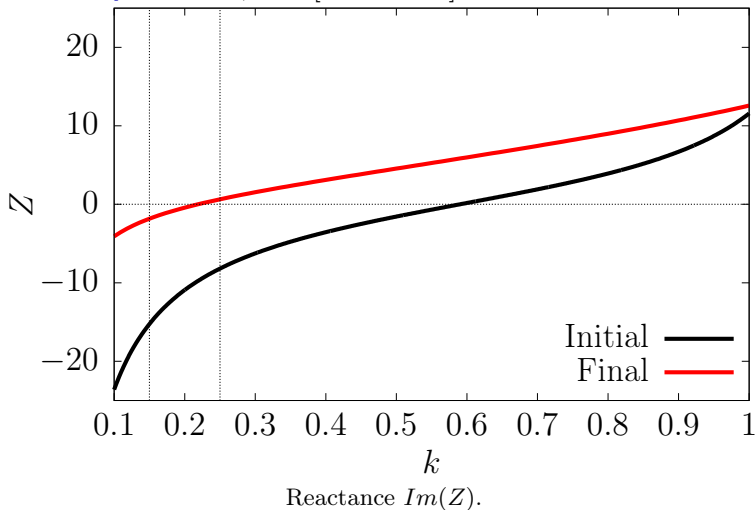


Converged.



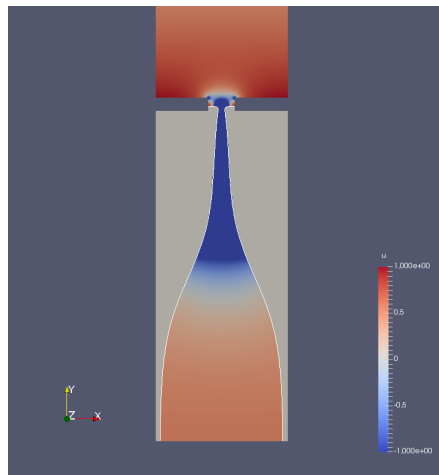
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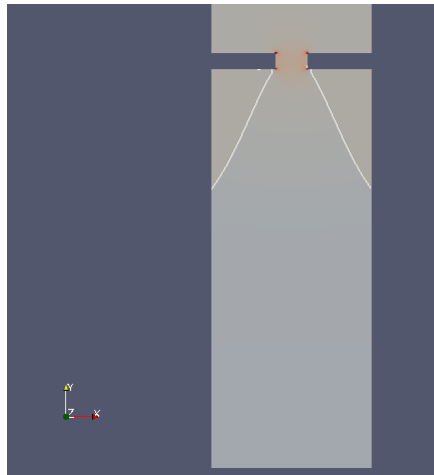


# Results

Viscosity:  $Z_T = 1.0$ ,  $k = [0.2 : 0.6]$



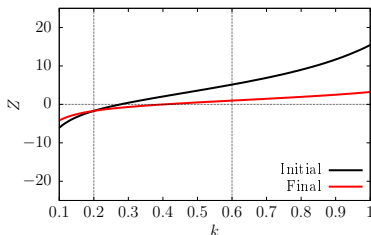
Initial condition.  
Coloured by  $\nabla_{\delta\theta}\mathcal{J}$ .



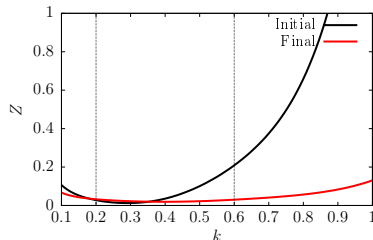
Converged.

# Results

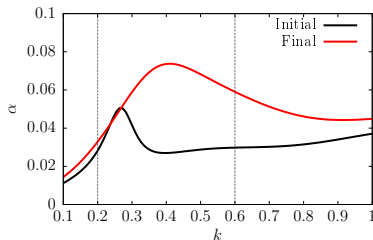
Viscosity:  $Z_T = 1.0$ ,  $k = [0.2 : 0.6]$



Reactance  $Im(Z)$ .



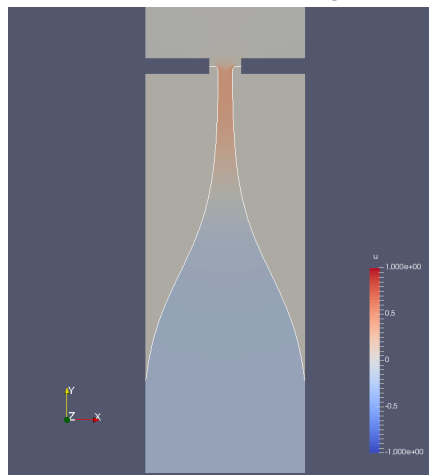
Resistance  $Re(Z)$ .



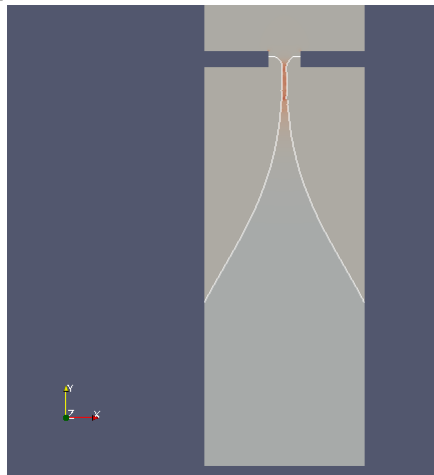
Absorption  $\alpha = 1 - |R|^2$ .

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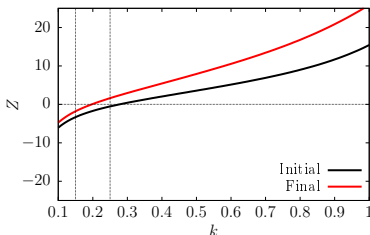
Initial condition.  
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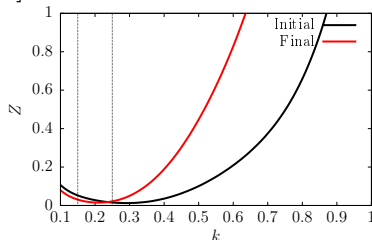
Converged.

# Results

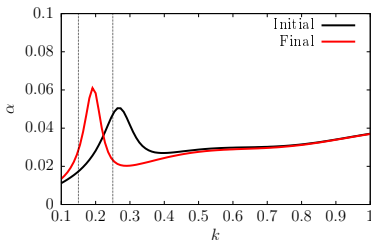
Viscosity:  $Z_T = 1.0$ ,  $k = [0.15 : 0.25]$



Reactance  $Im(Z)$ .



Resistance  $Re(Z)$ .



Absorption  $\alpha = 1 - |R|^2$ .

# Conclusion

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## Summary:

- Shape optimisation for impedance matching.
- Viscous model: compromise between Helmholtz and full linearised Navier-Stokes.

## Perspectives:

- Optimise from efficient/realistic configuration.
- Different cost functional (Penalty to initial guess, absorption, ...).

Thank you for your attention.